

Search method for an emission line of a GW background

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Abstract. A sharp emission line of a gravitational-wave background (GWB) would be an interesting observational target. Here we study an efficient method to detect a line GWB by correlating data of multiple GW detectors. We find that the width of frequency bin in the data analysis is a critical parameter, and, with the commonly-used value 0.25 Hz, the signal-to-noise ratio could be decreased by up to a factor of 6.6, compared with a finer width of 0.02 Hz. By reanalyzing the existing data with a smaller bin width, we might detect a precious line signal from the early universe.

1. Introduction

With the advent of the second-generation GW interferometric detectors, the first detection of a GW would be achieved in a couple of years. At present, the primordial GWs have not been detected yet. However, in the future, the high penetrating power of GWs could become quite advantageous to probe the early universe such as inflation, the subsequent reheating era, high energy physics, and possibly gravity theory beyond general relativity, and extend our observational reach before the big bang nucleosynthesis [1].

To maximize the scientific outputs from the accumulated data of detectors, we should deliberate on methods of data analysis and thoroughly search for stochastic GWBs, not only for given theoretical predictions but also in model independent manners. One of such GWs would be a sharp emission line of a cosmological GWB. The line GWB is not vital for the contemporary standard model of cosmology but is still allowed to exist. To create a line, there needs a sharply defined physical scale such as periodicity, boundaries, or discreteness. These could be realized in the cosmological scenarios with extra dimensions whose topologies are torus or warped between branes [2, 3], or in the massive gravity theories that gravitons have tiny mass [4].

This article is a short summary of our previous work [5], in which we have studied how GW emission lines are constrained by GW observations in a model-independent way. Throughout this paper, we use the unit $c = 1$ for the speed of light.

2. Doppler broadening and GW signals

We assume that a GWB is isotropic in the CMB rest frame but nearly monochromatic, having a sharp line spectrum. Seen from a detector moving relative to the CMB frame with velocity $\vec{v}(t)$, the GWB looks anisotropic because the observed frequency is Doppler-shifted, depending on the



propagation direction of a GW $\hat{\Omega}$. The relative velocity \vec{v}_s of the Solar-System barycenter to the CMB rest frame is toward the Galactic coordinate $(\ell, b) = (263.99^\circ \pm 0.14^\circ, 48.26^\circ \pm 0.03^\circ)$ with the magnitude $v_s = (1.230 \pm 0.003) \times 10^{-3}$ ($= 369.1 \text{ km s}^{-1}$) [6]. Below, for simplicity, we put $\vec{v}(t) = \vec{v}_s$, neglecting corrections (at most $\leq 10\%$) due to the velocity of the detector relative to the SSB. We also neglect minor relativistic effects and drop the terms of $O(|\vec{v}(t)|^2)$ for the Doppler effect. Then we have GW frequency in the observer's frame

$$f(t, \hat{\Omega}) = f_r [1 - v_s \cos \theta] , \quad (1)$$

with $\cos \theta = \hat{\Omega} \cdot \vec{v}_s / v_s$. Therefore, for a given observational frequency, the GW signals come from a ring on the sky (with a fixed θ). The total width of the Doppler broadening is given by $\delta f_D = 2f_r v_s \sim 0.2 (f_r / 100 \text{ Hz}) \text{ Hz}$.

We start with the correlation analysis for an anisotropic GWB (not specific to a line GWB) [7]. In the frequency space, the magnitude of a GWB is characterized conventionally by the energy density of GWs per logarithmic frequency bin per steradian divided by the critical energy density of the Universe, $\Omega_{\text{gw}}(f; \hat{\Omega})$. The correlation analysis is an efficient method to distinguish a GWB signal from detector noises (see *e.g.* [8–10]). With this method, we prepare data streams of two widely separated detectors with independent noises, and take cross correlations of their Fourier modes. After binning specified by the label k , its central frequency f_k , and its width δf_b ($> T^{-1}$, T : observation time, but much smaller than the all characteristic variation of signals), the expectation value of a signal in each bin k is given by [10]

$$\mu_k = \frac{3H_0^2}{8\pi^2} \frac{T \delta f_b}{f_k^3} Z(f_k). \quad (2)$$

Here we use the Hubble parameter $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and defined the integral

$$Z(f_k) \equiv \int \frac{d\hat{\Omega}}{4\pi} \Omega_{\text{gw}}(f_k; \hat{\Omega}) \gamma(f_k, \hat{\Omega}) , \quad (3)$$

and the direction-dependent overlap-reduction function

$$\gamma(f, \hat{\Omega}) \equiv \sum_A F_I^A(\hat{\Omega}) F_J^A(\hat{\Omega}) \exp \left[2\pi i f \hat{\Omega} \cdot \vec{d}_{IJ} \right] , \quad (4)$$

where $\vec{d}_{IJ}(t) \equiv \vec{X}_I(t) - \vec{X}_J(t)$ is the distance vector between the I th detector at the position $\vec{X}_I(t)$ and the J th detector at $\vec{X}_J(t)$. F_I^A is the antenna pattern function of the I th interferometer. So far the formulation is generic and we have not assumed any specific spectrum of a GWB.

Now we apply the above expressions specifically to the line GWB at a frequency f_r defined in the CMB frame. From Eq.(1), we can write the observed spectrum by

$$\Omega_{\text{gw}}(f; \theta) = \tilde{e}_{\text{gw}} f_r \delta[f - f_r(1 - v_s \cos \theta)], \quad (5)$$

where \tilde{e}_{gw} is the total energy of the line normalized by the critical density. For a line GWB, we define the overlap-reduction function $\Gamma(f, u)$ that is obtained by integrating $\gamma(f, \hat{\Omega})$ across the ring directions at $u \equiv \cos \theta = \text{constant}$ as

$$\Gamma(f, u) \equiv \int_0^{2\pi} \frac{d\phi}{2\pi} \gamma(f, \hat{\Omega}) . \quad (6)$$

From Eqs.(2)-(6), replacing the frequencies of slowly-varying functions with f_r and putting $\Gamma(f_k) \equiv \Gamma(f_r, u(f_k))$, we obtain

$$\mu_k \approx \frac{3H_0^2}{16\pi^2} \frac{T \tilde{e}_{\text{gw}}}{f_r^3 v_s} \delta f_b \Gamma(f_k) . \quad (7)$$

Note that the frequency f and the sky angle θ (equivalently u) are related by Eq. (1).

This correlation signal has three interesting behaviors (see [5] for more details). Firstly, $\Gamma(f)$ is in general a complex number and has a finite imaginary part. For an isotropic GWB (*e.g.* without a line component), the imaginary part of the correlated signal exactly cancels out due to the directional symmetry of GWs. In contrast, for a line GWB, the imaginary part does not cancel out because of the anisotropy induced by the Doppler effect. Secondly, at high frequency regime, the correlation signal is rapidly oscillating through the phase factor, $\exp[2\pi i f \hat{\Omega} \cdot \vec{d}_{IJ}]$ in $\Gamma(f_k)$, which strongly depends on the propagation direction $\hat{\Omega}$. The condition $f_r(\hat{\Omega} \cdot \vec{d}_{IJ}) \sim 1$ and Eq. (1) lead to the characteristic frequency interval δf_c for the oscillation, $\delta f_c \sim v_s/d_{IJ} \simeq 0.1(3000 \text{ km}/d_{IJ}) \text{ Hz}$. Comparing this result with the Doppler width, the number of oscillations in δf_D becomes larger for a higher frequency f_r . For another detector pair more separated than aLIGOs ($d_{IJ} \sim 3000 \text{ km}$), the characteristic interval δf_c becomes smaller. Thirdly, the frequency dependence of $\Gamma(f)$ changes in time due to the spin and revolution of the Earth because the relative direction from the GWB rest frame (the CMB frame) changes. These motions bring complications into the data analysis. However, they would be useful to distinguish true signals from artificial noise lines produced by instruments.

In Eq.(2), we assumed to take a small bin width δf_b so that the frequency dependence of relevant functions can be neglected within each bin. But this is not always the case. Indeed, the commonly used value by LIGO and VIRGO in [11] is $\delta f_b = 0.25 \text{ Hz}$, though the characteristic frequency interval is $\delta f_c \sim 0.1 \text{ Hz}$. For the case with $\delta f_b > \delta f_c$, the correlation signal $|\mu_k|$ averaged out by the integration of the wavy structure in each frequency bin. Then Eq.(7) should be replaced with the following integral

$$\mu_k = \frac{3H_0^2}{16\pi^2} \frac{T\tilde{e}_{\text{gw}}}{f_r^3 v_s} \int_{\delta f_b} df \Gamma(f). \quad (8)$$

3. Sensitivity to a line GWB

To calculate the signal-to-noise ratio (SNR) of the line detection, the noises of the two detectors are assumed to be independent and to have magnitudes much larger than those of the GW signals. Since a detector noise in the correlation signal is slowly varying function in frequency, we can use the same noise formula in [10] for both cases with smaller and larger bin than δf_c . Using Eqs. (7) and (8), the total squared SNR is evaluated as [5]

$$\rho^2 = \frac{9H_0^4}{32\pi^4} \frac{T\tilde{e}_{\text{gw}}^2 \delta f_b}{f_r^6 v_s^2 P_I(f_r) P_J(f_r)} \sum_k |\Gamma(f_k)|^2 \quad \text{for } \delta f_b \ll \delta f_c, \quad (9)$$

$$\rho^2 = \frac{9H_0^4}{32\pi^4} \frac{T\tilde{e}_{\text{gw}}^2 \delta f_b}{f_r^6 v_s^2 P_I(f_r) P_J(f_r)} \sum_k \frac{1}{(\delta f_{b(k)})^2} \left| \int_{\delta f_{b(k)}} df \Gamma(f) \right|^2 \quad \text{for } \delta f_b \gg \delta f_c. \quad (10)$$

with the one-sided noise spectra $P_I(f)$. For a total observational duration T much longer than a day, the factors involving with $\Gamma(f_k)$ oscillate diurnally. So the factors should be regarded as time averaged one.

In Fig. 1, the sensitivity to the normalized energy density \tilde{e}_{gw} is shown as a function of the line frequency f_r . We assumed a 1yr observation with two aLIGO detectors at the detection threshold of $\rho = 10$. For their noise spectra $P_{I,J}(f)$, we use the fitting formula given in [12]. For our demonstration here, we selected the following four widths; 0 Hz, 0.1 Hz, 0.25 Hz, and 2.5 Hz. The first one means a sufficiently small width (still satisfying $\delta f_b > T^{-1}$). Figure 1 shows that the sensitivity decreases significantly for larger bin widths due to the cancellation of the wavy structure. The degradation factors from $\delta_b = 0 \text{ Hz}$ case, say, at $f_r = 70 \text{ Hz}$, is 6.8 for

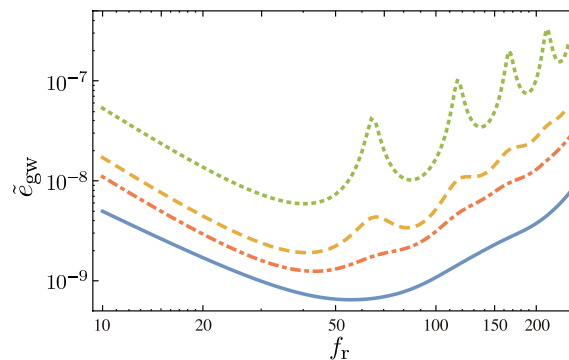


Figure 1. Sensitivity to the normalized energy density \tilde{c}_{gw} of a line GWB, assuming 1 yr observation with two aLIGOs at detection threshold $\rho = 10$. The blue (solid) curve is for the bin width $\delta_b = 0$ Hz. The other curves from the top are with the bin width 2.5 Hz (green, dotted), 0.25 Hz (orange, dashed), 0.1 Hz (red, dotted-dashed), respectively. Adopted from [5].

$\delta f_b = 0.25$ Hz and 22 for 2.5 Hz. We also examined bin widths smaller than $\delta f_c = 0.1$ Hz. For a line GWB with $10 \text{ Hz} < f_r < 600 \text{ Hz}$, the degradations (compared with $\delta f_b = 0$) are within 1.1 for $\delta f_b = 0.01$ Hz, 1.2 for 0.02 Hz, 1.5 for 0.04 Hz, 3.7 for 0.1 Hz and 7.9 for 0.25 Hz. Therefore, to realize a high SNR, we should set the bin width sufficiently smaller than the characteristic interval $\delta f_c \sim 0.1$ Hz.

4. Conclusions

We have studied the detector signals from a line GWB and suggested the correlation analysis method for efficiently searching for such GW signals. Since the overlap reduction function has fine wavy structures, the correlation signal could be canceled out unless the bin width δf_b is much smaller than the characteristic frequency interval $\delta f_c \sim 0.1$ Hz. Nevertheless, in the standard correlation analysis of GWBs, the commonly used width is 0.25 Hz. By reanalyzing the existing data with a smaller width, we might actually uncover an important signal from the early universe.

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